

Study Guide for the Midterm Exam
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The Midterm Exam will cover material from the Appendix F, Sections 3.2 – 10.1.

Appendix F (Review of Chapters 4 & 5)

1. Perform operations on polynomials – add, subtract, and multiply.

- To add – combine like terms. (Combine like terms – add coefficients of like variable expressions and leave the variable the same.)
- To subtract, change the sign of each term in the polynomial following the subtraction sign. Combine like terms.
- To multiply, use the distributive property or FOIL.

Some examples:

$$(x^2 - 2x + 1) - (2x^2 + 3x - 4) = x^2 - 2x + 1 - 2x^2 - 3x + 4 = -x^2 - 5x + 6$$

$$(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

$$(x+2)(x^2 - 2x + 4) = x^3 - 2x^2 + 4x + 2x^2 - 4x + 8 = x^3 + 8$$

2. Factor polynomials – binomials, trinomials, and four-term polynomials.

- To factor, first remove the GCF from the polynomial.
- Identify the number of terms in the polynomial.
- If there are two terms, determine if the polynomial fits one of the following patterns and apply as necessary.
 - Difference of Two Squares $a^2 - b^2 = (a + b)(a - b)$
 - Difference of Two Cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 - Sum of Two Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
 - Sum of Two Squares – prime
- If there are three terms, then find the two binomials whose FOIL is the given trinomial.
 - If the leading coefficient (coefficient of squared term) is 1, then find the factors of the constant term whose sum or difference is the middle coefficient. If the last sign is a sum, then the signs of the binomial factors are the same as whatever sign is on the middle term. If the last sign is a

difference, then the signs of the binomial factors are different. The sign of the middle term determines the sign of the larger number.

- If the leading coefficient is not 1, then use the grouping method to factor. Find the product of the leading coefficient and the constant term. Then find the factors of this number whose sum/difference is the middle coefficient. Replace the middle term with these two terms and use grouping to factor.

- If there are four terms, use grouping to factor.
- Always check by factoring.

Some examples:

$$4b^{10} - 24b^9 - 160b^8 = 4b^8(b^2 - 6b - 40) = 4b^8(b - 10)(b + 4)$$

$$4s^2 + 9s + 5 = 4s^2 + 4s + 5s + 5 = 4s(s + 1) + 5(s + 1) = (s + 1)(4s + 5)$$

$r^2 + 36$ is prime.

$$b^3 - 64 = (b)^3 - (4)^3 = (b - 4)(b^2 + 4b + 64)$$

$$125r^3 + 216g^3 = (5r)^3 + (6g)^3 = (5r + 6g)(25r^2 - 30rg + 36g^2)$$

$$r^4 - 2401 = (r^2 - 49)(r^2 + 49) = (r - 7)(r + 7)(r^2 + 49)$$

3. Apply properties of exponents.

- If the exponential expressions are multiplied and the bases are the same, keep the base and add the exponents.
- If the exponential expression is raised to a power, keep the base and multiply the exponents.
- If a product is raised to a power, apply the exponent to each factor and simplify.
- If the exponential expressions are divided and the bases are the same, keep the bases and subtract the exponents.
- If a quotient is raised to a power, apply the exponent to the numerator and denominator and simplify.
- Apply the definition of a negative exponent: A negative power indicates reciprocal. So, take the reciprocal of the base and raise this to the positive power.
- Apply the definition of the zero exponent: A base raised to the 0 power is 1.

Some examples:

$$\left(\frac{x^{-4}y}{z^3}\right)^{-3} = \frac{x^{12}y^{-3}}{z^{-9}} = \frac{x^{12}z^9}{y^3}$$

$$(3t)^9(3t)^{-5} = (3t)^4 = 81t^4$$

$$\frac{(m^3n)^{-7}}{m^{-15}n^5} = \frac{m^{-21}n^{-7}}{m^{-15}n^5} = m^{-21-(-15)}n^{-7-5} = m^{-6}n^{-12} = \frac{1}{m^6n^{12}}$$

Section 3.2

1. Find solutions to linear equations in two variables.

- A solution is an ordered pair that makes the equation true.
- If one value of the ordered pair is given, substitute the given value and solve for the unknown value.

Example:

Find the solution for $x + 2y = 3$ if $x = -1$.

Solution:

Replace x with -1 to get $-1 + 2y = 3$ and solve for y .

$$-1 + 2y + 1 = 3 + 1 \quad (\text{Add 1 to both sides to isolate variable.})$$

$$2y = 4 \quad (\text{Simplify.})$$

$$y = 2 \quad (\text{Divide both sides by 2.})$$

The solution is $(-1, 2)$.

$$\text{Check: } -1 + 2(2) = 3$$

$$-1 + 4 = 3$$

$$3 = 3$$

Since the ordered pair makes the equation true, the solution is correct.

2. Find the x and y intercepts of linear equations.

- The x -intercept is the point where the graph crosses the x -axis. This point is always of the form (some number, 0). To find the x -intercept, substitute 0 for y and solve for x . Write the x -intercept as $(\#, 0)$.
- The y -intercept is the point where the graph crosses the y -axis. This point is always of the form (0, some number). To find the y -intercept, substitute 0 for x and solve for y . Write the y -intercept as $(0, \#)$.

Example: Find the x and y intercepts of the graph of $2x - y = 4$.

Solution: To find the x -intercept, replace y with 0 to get

$$2x - 0 = 4$$

$$2x = 4$$

$$x = 2$$

The x -intercept is $(2, 0)$.

To find the y -intercept, replace x with 0 to get

$$2(0) - y = 4$$

$$-y = 4$$

$$y = -4$$

The y-intercept is (0, -4).

3. Graph a linear equation.

- To graph a linear equation, you must find at least two ordered pairs that are solutions to the equation.
 - Find two ordered pairs by assigning x any two values and finding the corresponding y value.
 - Find the x and y intercepts for two points on the graph.

Example: Graph the equation $x + y = 4$.

Solution: To graph by plotting points, solve for y and complete a table of solutions.

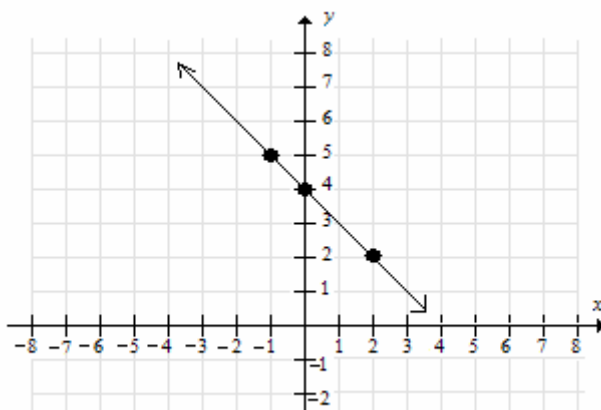
$$x + y = 4$$

$$y = -x + 4 \quad \text{(Subtract x from both sides.)}$$

Find three solutions by selecting three values of x (a negative value, zero, and a positive value). Substitute the selected x-values to solve for the corresponding y-values as shown.

X	$y = -x + 4$	(x, y)
-1	$y = -(-1) + 4 = 5$	(-1, 5)
0	$y = -(0) + 4 = 4$	(0, 4)
2	$y = -(2) + 4 = 2$	(2, 2)

Plot the three solutions from the table and then connect the points to show the complete solution set to $y = -x + 4$ or $x + y = 4$.



4. Recognize and graph horizontal and vertical lines.

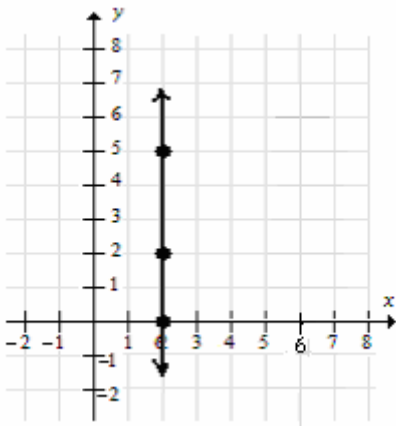
- The equation of a horizontal line is always $y = b$, where b is any real number. To graph this line, plot the point $(0, b)$ and draw the horizontal line through this point. (Remember horizontal goes from left to right.)
- The equation of a vertical line is always $x = a$, where a is any real number. To graph this line, plot the point $(a, 0)$ and draw the vertical line through this point. (Remember vertical goes from bottom to top.)

Example: Graph the equation $x = 2$.

Solution: The only ordered pairs that satisfy this equation are those whose x -coordinate is 2. Some solutions are

x	y
2	0
2	2
2	5

The graph of the equation is the vertical line through $x = 2$.



5. Use linear models to solve application problems.

- A model is just an equation that has some meaning to the x and y values. When solving application problems, first make sure that you know the meaning of the two variables. Then substitute the given value into the equation and solve for the remaining variable.

Example: The equation $y = 138.41x + 2961.8$ models the average undergraduate cost of attending a public 2-year college x years after 1986. Use this model to find the cost of attending a 2 year college in 1996. (Source: <http://nces.ed.gov/fastfacts>)

Solution: The year 1996 is 10 years after 1986, so assign x the value 10.

$$y = 138.41(10) + 2961.8 = 1384.1 + 2961.8 = 4345.9$$

In 1996, the average undergraduate cost of attending a public 2-year college was \$4345.90.

Section 3.3

1. Find the slope of a line given two points.

- To find the slope given two points, label one of the points as (x_1, y_1) and the other point as (x_2, y_2) .
- Substitute these values into the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Remember that slope is a numerical measure for the steepness of a line.
 - If the slope is positive, the line is increasing from left to right.
 - If the slope is negative, the line is decreasing from left to right.
 - If the slope is zero, the line is horizontal.
 - If the slope is undefined, the line is vertical.
- Slope is the ratio of change in y to change in x . In other words, it determines how you move to another point on a line. Think of the slope as rise over run. The numerator tells you how far to move up (if slope is positive) or down (if slope is negative) between two points and the denominator tells you how far to move right.

Example:

Find the slope of the line containing the points $(5, -2)$ and $(-3, 4)$.

Solution:

Let $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (-3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{-3 - 5} = \frac{6}{-8} = -\frac{3}{4}$$

2. Find the slope of a line given an equation.

- The slope is determined from an equation only if the equation is solved for y and in the form $y = mx + b$. The slope is the coefficient of the variable x .
- Note: The point $(0, b)$ is the y -intercept of the graph of the line.

Example: Find the slope of the line $2x - y = 4$.

Solution: To find the slope, we need to write the equation in the slope-intercept form. This gives us

$$2x - y = 4$$

$$-y = -2x + 4 \quad (\text{Subtract } 2x \text{ from both sides.})$$

$$y = 2x - 4 \quad (\text{Divide both sides by } -1.)$$

Since the coefficient of x is 2, the slope of the line is 2.

3. Determine if lines are parallel or perpendicular.

- To determine if lines are parallel or perpendicular, you must first find the slope of each line (solve both equations for y and compare the coefficients of x).
- If the slopes are the same, then the lines are parallel.
- If the slopes are negative reciprocals, then the lines are perpendicular.
- If the slopes do not relate in either of the above ways, then the lines are neither parallel nor perpendicular.

Example: Determine if the lines given below are parallel, perpendicular or neither.

$$2x - 3y = 6$$

$$3x + 2y = 12$$

Solution: We need to find the slope of each line by writing each in the slope-intercept form.

$$2x - 3y = 6$$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$

$$m = \frac{2}{3}$$

$$3x + 2y = 12$$

$$2y = -3x + 12$$

$$y = -\frac{3}{2}x + 6$$

$$m = -\frac{3}{2}$$

Since these slopes are negative reciprocals, these lines are perpendicular.

4. Given a slope, match its corresponding graph.

- Use the facts below:
 - If the slope is positive, the line is increasing from left to right.
 - If the slope is negative, the line is decreasing from left to right.
 - If the slope is zero, the line is horizontal.
 - If the slope is undefined, the line is vertical.
 - Also, check rise and run of the graph to see if it matches the value of the slope.
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5. Write an equation in slope-intercept form.

- An equation is written in slope-intercept form if it solved for y and in the form $y = mx + b$.
- The value of m is the slope and the point (0, b) is the y-intercept.
- A line can be graphed using this information. Begin at the point (0, b) and use the slope to move to another point.

Example: Find the slope and y-intercept of the equation $2x - 3y = 12$.

Solution: Solve the equation for y. Subtract $2x$ from both sides to get $-3y = -2x + 12$. Now divide both sides by -3 to get $y = \frac{2}{3}x - 4$.

The slope is the coefficient of x which is $\frac{2}{3}$. The y-intercept is the point (0, -4).

Note: To graph with this information, plot the beginning point (0, -4). From this point, move up 2 units and right 3. Graph the line between these points.

6. Use a point and a slope to graph a line.

- Plot the given point. From this point, use the slope to rise and run to the second point.
- Draw the line through these two points.

Example: Draw the graph of the line $y = -\frac{2}{3}x + 2$.

Solution: The slope of the line is $-2/3$ and the y-intercept is (0, 2). Plot the y-intercept of (0, 2). From this point, move down 2 units and right 3 units. This will take you to the point (3, 0). Now, draw the line through (0, 2) and (3, 0).

Section 3.4

1. Write the equation of a line given a point and a slope.

- If the point is the y-intercept point, then you already know the information to get the equation. Substitute the given slope for the value m and the value of b from the point (0, b) in the slope-intercept form $y = mx + b$.
- If the point given is not the y-intercept, then use the point-slope form to write the equation of the line. The point-slope form is $y - y_1 = m(x - x_1)$.
 - Substitute the given point for (x1, y1).
 - Use the distributive property to remove parentheses from the right side.
 - If there are fractions, multiply both sides by the LCD to clear fractions.
 - Solve the equation for y to get in the form $y = mx + b$.

- Check your equation by plugging in given point to see if it makes the equation true. (You can also check on the calculator – see if given point is on the graph or in the table.)
- **ALTERNATE METHOD:** You can also use the slope-intercept form of the line to find your equation. You will first need to find the slope. Once you have a slope, use this for the value of m and one of the given points for the values (x, y) and plug into $y = mx + b$ to find the value of b .

Example: Find the equation of the line through $(2, 5)$ with slope $m = 4$.

Solution: Use the point-slope form with $(x_1, y_1) = (2, 5)$ and $m = 4$.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 4(x - 2)$$

$$y - 5 = 4x - 8$$

$$y = 4x - 3$$

ALTERNATE SOLUTION: Let $m = 4$ and $(x, y) = (2, 5)$.

$$y = mx + b$$

$$5 = 4(2) + b$$

$$5 = 8 + b$$

$$-3 = b$$

The equation is $y = 4x - 3$.

2. Write the equation of a line given two points.

- The first step is to find the slope of the line. Substitute the two points into the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ and simplify.
- Substitute the slope and one of the given points into the point-slope form $y - y_1 = m(x - x_1)$ and solve for y .

Example: Find the equation of the line through $(1, -4)$ and $(-3, 2)$.

Solution:

Step One: Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{-3 - 1} = \frac{6}{-4} = -\frac{3}{2}$$

Step Two: Use the point-slope form with one of the points and the slope.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{2}(x - (-3))$$

$$y - 2 = -\frac{3}{2}(x + 3)$$

$$y - 2 = -\frac{3}{2}x - \frac{9}{2}$$

$$2y - 4 = -3x - 9 \quad \text{Multiply both sides by 2 to clear fractions.}$$

$$2y = -3x - 5$$

$$y = -\frac{3}{2}x - \frac{5}{2}$$

3. Write the equation of a line given a point and a line either parallel or perpendicular to it.

- The first step is to find the slope of the line. Find the slope of the given line. If the lines are parallel, then use the same slope in the point-slope form. If the lines are perpendicular, use the negative reciprocal in the point-slope form.
- Substitute the slope and the given point in the point-slope form $y - y_1 = m(x - x_1)$ and solve for y.

Example: Find the equation of the line that goes through (4, 7) and is parallel to $y = \frac{1}{2}x + 3$.

Solution:

Step One: Find the slope.

The given equation is in the slope-intercept form, so the slope is $\frac{1}{2}$. Since the lines are parallel, the slopes are the same. Therefore, the slope of our equation is $\frac{1}{2}$.

Step Two: Use the point-slope form with the point (4, 7) and the slope of $\frac{1}{2}$.

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{1}{2}(x - 4)$$

$$y - 7 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x + 5$$

Section 7.1

1. Find the midpoint between two points.

- Use the midpoint formula to calculate the midpoint between two ordered pairs. Plot the given points, draw the line segment between the points, and plot the midpoint. (See Midpoint Worksheet.)
- The midpoint formula is $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. The midpoint is the average of the x-values and the average of the y-values.

Example: Find the midpoint between (5, -2) and (-3, 6).

Solution: Use the formula with $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (-3, 6)$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + (-3)}{2}, \frac{-2 + 6}{2}\right) = \left(\frac{2}{2}, \frac{4}{2}\right) = (1, 2)$$

Section 7.2

1. Applications on linear equations

- Given two data points, write the equation that models the real-life situation. This involves finding the slope and then using the point-slope form to get the equation.

Example: Suppose the enrollment of GPC was 16,000 in 1998 and 22,000 in 2004. If x is the number of years after 1998 and y is the GPC enrollment, use the points (0, 16000) and (6, 22000) to write a linear equation that represents GPC enrollment x years after 1998.

Solution: First find the slope.

$$\text{The slope is } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{22000 - 16000}{6 - 0} = \frac{6000}{6} = 1000.$$

Now, use the point-slope form with one of the points and the slope to find the equation.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - 16000 &= 1000(x - 0) \\y - 16000 &= 1000x + 0 \\y &= 1000x + 16000\end{aligned}$$

Section 7.3

1. Know the definition of a function.

- A function is a rule that assigns only one output for each input. (Example: social security number and the person assigned to this number)

2. Use function notation.

- To evaluate a function using function notation, substitute the given x-value into the function and simplify to find the corresponding y value. Remember that $f(x)$ is another way to represent y .

Example: Find $f(-3)$ if $f(x) = x^2 + 2x - 1$.

Solution: $f(-3) = (-3)^2 + 2(-3) - 1 = 9 - 6 - 1 = 2$

3. Rewrite an equation using function notation.

- Solve the equation for y and replace y with $f(x)$.

Example: Write the equation $4x - y = 8$ in function notation.

Solution: To write in function notation, solve the equation for y and then replace y with $f(x)$.

$$\begin{aligned} 4x - y &= 8 \\ -y &= -4x + 8 \\ y &= 4x - 8 \end{aligned}$$

So, $f(x) = 4x - 8$.

Sections 8.1, 8.2, and 8.3

1. Verify that a given ordered pair is a solution to a system of equations.

- Substitute the given point into both equations. For this point to be a solution, it must make both equations true.

Example: Is $(4, -1)$ a solution to the system containing the equations $3x - y = 13$ and $y = -2x - 9$?

Solution: Substitute the ordered pair into both equations and determine if the resulting statements are true.

$$\begin{aligned} 3x - y &= 13: 3(4) - (-1) = 13 \\ 12 + 1 &= 13 \\ 13 &= 13 \quad \text{True} \end{aligned}$$

$$\begin{aligned} y &= -2x - 9: -1 = -2(4) - 9 \\ -1 &= -8 - 9 \\ -1 &= -17 \quad \text{False} \end{aligned}$$

Since the ordered pair does not make both equations true, it is NOT a solution to the system.

2. Find the solution to a system by graphing, using substitution, or elimination.

- A system will have one of three possible solutions – one point (if two lines intersect), no solution (if two lines are parallel), or infinitely many solutions (if the two lines are coinciding). Two lines will intersect if their slopes are different. Two lines are parallel if their slopes are the same and their y-intercepts different. Two lines are coinciding if their slopes and y-intercepts are the same.
- To solve by graphing, graph both lines using 1) the intercepts, 2) any two points, or 3) the slope and y-intercept. From the graph, determine the point of intersection. Write the solution as this ordered pair.
- To solve by substitution, solve one equation for one of the variables. Substitute this expression into the other equation for this variable and solve the resulting equation. This will give you the value of one of the coordinates. To find the other value, substitute this number into one of the equations and solve.
- To solve by elimination, the goal is to add the two equations so that one of the variables is eliminated. For this to happen, one of the variables must have opposites as coefficients (like 3 and -3). If the equations are not in this form initially, multiply one or both equations by some number to make the coefficients of one of the variables opposite. Then add the equations and solve the resulting equation. Once one value is known, plug into one of the equations to find the other value.

Example: Solve the system $2x - y = 3$ and $x + 6y = 8$ by substitution and elimination.

Solution:

SUBSTITUTION:

Solve $x + 6y = 8$ for x . This gives us $x = 8 - 6y$. Substitute this expression into the other equation in place of x and solve for y .

$2x - y = 3$	
$2(8 - 6y) - y = 3$	
$16 - 12y - y = 3$	Distribute to clear parentheses.
$16 - 13y = 3$	Combine like terms.
$-13y = -13$	Subtract 16 from both sides.
$y = 1$	Divide both sides by -13.

Now solve for x : $x = 8 - 6y = 8 - 6(1) = 8 - 6 = 2$. So, the solution is (2, 1).

ELIMINATION:

To eliminate the x terms from the equations, we must find a number that both 2 and 1 divide into evenly. This number is 2. Therefore, we need coefficients of 2 and -2 on the x terms in the equations. Since the first equation has a 2 on the coefficient of x , we just need to get a -2 for the coefficient of x in the 2nd equation. To do this, multiply the 2nd equation by -2 (-2 divided by 1, that is, the number we need divided by the given coefficient).

This gives us the new system:

$$\begin{array}{rcl} 2x - y = 3 & \text{or} & 2x - y = 3 \\ -2(x + 6y = 8) & & -2x - 12y = -16 \end{array}$$

Adding the two equations together, we get $-13y = -13$, or $y = 1$.

Now solve for x by replacing the y variable with 1 in one of the two original equations.

$2x - 1 = 3$ or $2x = 4$ or $x = 2$. So, the solution is $(2, 1)$.

3. Determine the number of solutions to a system – none, one, or infinitely many.

- Use the slope and y-intercept to determine this information. To get this information, solve both equations for y .
- If the slopes of the lines are different, then the lines intersect and therefore the system has one solution.
- If the slopes are the same and their y-intercepts different, then the lines are parallel and therefore the system has no solution.
- If the slopes and y-intercepts are the same, then the lines are coinciding and therefore the system has infinitely many solutions.

See example below for this concept.

4. Determine if a system is consistent, inconsistent, or has dependent equations.

- Consistent – intersecting lines (one solution)
- Inconsistent – parallel lines (no solution)
- Dependent – coinciding lines (infinitely many solutions)

Example: For the system below, determine how many solutions it has and what type of system it is.

$$\begin{array}{l} 3x - y = 6 \\ 6x - 2y = 9 \end{array}$$

Solution: Solve both equations for y and compare their slopes and y-intercepts.

1 st Equation: $3x - y = 6$ $-y = -3x + 6$ $y = 3x - 6$ Slope = 3 y-intercept = $(0, -6)$	2 nd Equation: $6x - 2y = 9$ $-2y = -6x + 9$ $y = 3x - 9/2$ Slope = 3 y-intercept = $(0, -9/2)$
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Since the slopes are the same and the y-intercepts are different, these lines are parallel. Therefore, the system has no solution and is inconsistent.

5. Use your calculator to find the intersection point.

- Solve both equations for y.
- Press Y= and input the two equations into Y1 and Y2.
- Graph the lines by pressing GRAPH.
- To find the point of intersection, press 2nd TRACE and choose option 5 (Intersect). Press ENTER for First Equation. Press ENTER for Second Equation. For Guess, move the cursor left or right to the point of intersection and press ENTER.
- The point of intersection should be displayed at the bottom of the screen.

Section 8.5

1. Solve applications of systems.

- Define two unknown quantities and two equations that relate these quantities and then use one of the methods from 8.1 – 8.3 to solve the system.

Example: Tickets to a production of King Lear at the College of DuPage cost \$5 for general admission or \$4 with a student ID. If 184 people paid to see a performance and \$812 was collected, how many student tickets were sold?

Solution:

Let g = the number of general admission tickets sold.

Let s = the number of student tickets sold.

To write the system, use the facts that the total number of tickets sold was 184 and the total collected was \$812. This gives us

$$\begin{aligned}g + s &= 184 \\ 5g + 4s &= 812\end{aligned}$$

Now solve the system with either substitution or elimination. To use elimination, multiply the 1st equation by -5 to eliminate the g terms from the equations. This gives us the new system

$$\begin{aligned}-5g - 5s &= -920 \\ 5g + 4s &= 812\end{aligned}$$

Adding the equations we get

$$\begin{aligned}-s &= -108 \\ s &= 108\end{aligned}$$

Since this solved for the value of s , we have the answer to our question. The number of student tickets sold was 108.

Section 10.1 Roots and Radicals

1. Evaluate the n th root of a number.

The n th root of the number is the number that must be raised to the n th power to get back to the original number.

$$\sqrt[n]{a} = b \text{ if } b^n = a$$

Some examples: $\sqrt{81} = 9$, $\sqrt[3]{-8} = -2$, $\sqrt[4]{625} = 5$, $\sqrt[5]{32} = 2$

2. Determine if the root of a number is rational, irrational or not a real number.

If the index (value of n) is even and a is negative, then $\sqrt[n]{a}$ is not a real number. Examples: $\sqrt{-4}$, $\sqrt{-100}$, $\sqrt[4]{-16}$

If the radicand (value of a) is a perfect power of the index, then $\sqrt[n]{a}$ is a rational number.

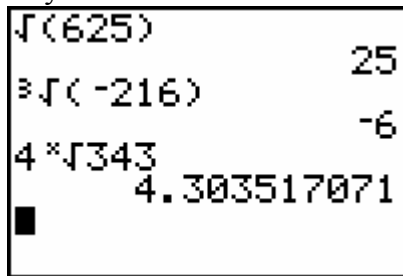
Examples: $\sqrt{25}$, $\sqrt[3]{27}$, $\sqrt[4]{81}$, $\sqrt{\frac{49}{64}}$

If the radicand is not a perfect power of the index and the root is a real number, then $\sqrt[n]{a}$ is irrational.

Examples: $\sqrt{5}$, $\sqrt[3]{12}$, $\sqrt[4]{8}$

3. Use your calculator to find roots.

Keystrokes on calculator:



2^{nd} , x^2 , 625,), Enter

MATH, 4, 216,), Enter

4, MATH, 5, 343, Enter

(The 4 represents the index. If the index is greater than 3, you must type this first before you select the root key.)

4. Apply the inverse properties of roots.

$$\left(\sqrt[n]{a}\right)^n = a, \text{ if } \sqrt[n]{a} \text{ is defined.}$$

Examples:

$$\left(\sqrt{5}\right)^2 = 5$$

$$\left(\sqrt{x+1}\right)^2 = x+1 \text{ (provided } x+1 \geq 0)$$

$$\left(\sqrt[3]{-8}\right)^3 = -8$$